

# Ghost contributions to charmonium production in polarized high-energy collisions

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## Abstract

In a previous paper [Phys. Rev. D **68**, 034017 (2003)], we investigated the inclusive production of prompt  $J/\psi$  mesons in polarized hadron-hadron, photon-hadron, and photon-photon collisions in the factorization formalism of nonrelativistic quantum chromodynamics providing compact analytic results for the double longitudinal-spin asymmetry  $\mathcal{A}_{LL}$ . For convenience, we adopted a simplified expression for the tensor product of the gluon polarization four-vector with its charge conjugate, at the expense of allowing for ghost and anti-ghosts to appear as external particles. While such ghost contributions cancel in the cross section asymmetry  $\mathcal{A}_{LL}$  and thus were not listed in our previous paper, they do contribute to the absolute cross sections. For completeness and the reader's convenience, they are provided in this addendum.

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The factorization formalism of nonrelativistic QCD (NRQCD) [1] provides a rigorous theoretical framework for the description of heavy-quarkonium production and decay. This formalism implies a separation of process-dependent short-distance coefficients, to be calculated perturbatively as expansions in the strong-coupling constant  $\alpha_s$ , from supposedly process-independent long-distance matrix elements (MEs), to be extracted from experiment, and takes into account the complete structure of the  $Q\bar{Q}$  Fock space, which is spanned by the states  $n = {}^{2S+1}L_J^{(C)}$  with definite spin  $S$ , orbital angular momentum  $L$ , total angular momentum  $J$ , and color multiplicity  $C = 1, 8$ . By velocity scaling rules, the MEs are predicted to scale with a definite power of the heavy-quark ( $Q$ ) velocity  $v \ll 1$ , so that a small number of these non-perturbative parameters should allow for meaningful predictions in practice.

In Ref. [2], we applied the NRQCD factorization formalism to the inclusive production of prompt  $J/\psi$  mesons in polarized hadron-hadron, photon-hadron, and photon-photon collisions and provided compact analytic results for the double longitudinal-spin asymmetry  $\mathcal{A}_{LL}$ , defined in Eq. (2.1) of Ref. [2]. Specifically, we considered inclusive  $J/\psi$  production in polarized  $pp$ ,  $\gamma d$ , and  $\gamma\gamma$  collisions, appropriate for the RHIC-Spin experiments at the BNL Relativistic Heavy Ion Collider (RHIC), the SLAC fixed-target experiment E161, and the TeV-Energy Superconducting Linear Accelerator (TESLA) operated in the  $e^+e^-$  and  $\gamma\gamma$  modes, respectively. We took the  $J/\psi$  mesons to be unpolarized.

There is a technical subtlety related to the definition of the polarization four-vector  $\varepsilon(p, \xi)$  of an external gluon, with four-momentum  $p$  and helicity quantum number  $\xi = \pm 1$ , which is potentially prone to create confusion. As for the tensor product of  $\varepsilon(p, \xi)$  with its charge conjugate, a natural choice, which avoids the introduction of unphysical degrees of gluon polarization, is

$$\varepsilon_\mu(p, \xi)\varepsilon_\nu^*(p, \xi) = \frac{1}{2} \left( -g_{\mu\nu} + \frac{p_\mu\eta_\nu + p_\nu\eta_\mu}{k \cdot \eta} + i\xi\epsilon_{\mu\nu\rho\sigma} \frac{p^\rho\eta^\sigma}{p \cdot \eta} \right), \quad (1)$$

where  $\eta$  is an arbitrary light-like four-vector orthogonal to  $p$ , with  $\eta^2 = 0 \neq p \cdot \eta$ . An obvious disadvantage of Eq. (1) is that it introduces a host of terms involving  $\eta$  in intermediate results. In practical calculations such as the one performed in Ref. [2], it is therefore advantageous to omit the second term on the right-hand side of Eq. (1) and to identify  $\eta$  with the four-momentum  $p'$  of another external parton [3], so that

$$\varepsilon_\mu(p, \xi)\varepsilon_\nu^*(p, \xi) = \frac{1}{2} \left( -g_{\mu\nu} + i\xi\epsilon_{\mu\nu\rho\sigma} \frac{p^\rho p'^\sigma}{p \cdot p'} \right), \quad (2)$$

at the expense of endowing the gluon with unphysical degrees of polarization, which must be eliminated by subtracting contributions arising from the presence of its ghost  $h$  and anti-ghost  $\bar{h}$  as external particles. Since such ghost contributions cancel in the cross section differences appearing in the numerator of  $\mathcal{A}_{LL}$ , as illustrated below, we did not list them in Ref. [2]. However, they are necessary to recover the well-known expressions for the unpolarized cross sections entering the denominator of  $\mathcal{A}_{LL}$ , as we did. In this sense, they were included in our numerical analysis.

Recently, there has been renewed interest in charmonium production by polarized hadron-hadron and photon-hadron collisions [4, 5]. In Ref. [4], the  $J/\psi$  and  $\psi'$  polarizations were predicted for polarized  $pp$  collisions at RHIC-Spin. In Ref. [5], the squares of the helicity amplitudes  $\mathcal{M}(a, b, c)$  of the partonic subprocesses  $\gamma(a) + g(b) \rightarrow Q\bar{Q}[n] + g(c)$  and  $g(a) + g(b) \rightarrow Q\bar{Q}[n] + g(c)$  were listed for  $n = {}^1S_0^{(C)}, {}^3S_1^{(C)}, {}^1P_1^{(C)}, {}^3P_J^{(C)}$  with  $J = 0, 1, 2$  and  $C = 1, 8$ . The longitudinally-polarized differential cross sections evaluated from these helicity amplitudes were found to agree with our results [2] after properly subtracting the ghost contributions mentioned above, which we had provided to the authors of Ref. [5] via private communication. Since these contributions may be useful for applications by other authors as well, we decided to publish them in this addendum to Ref. [2].

In the following, we present the differential cross sections  $d\sigma/dt$  of the partonic subprocesses

$$\{\gamma, g\}h \rightarrow c\bar{c}[n]h. \quad (3)$$

Here and in the following,  $s$ ,  $t$ , and  $u$  denote the usual Mandelstam variables. The results for  $\{\gamma, g\}\bar{h} \rightarrow c\bar{c}[n]\bar{h}$  are identical by charge-conjugation invariance, while those for  $h\{\gamma, g\} \rightarrow c\bar{c}[n]h$  and  $h\bar{h} \rightarrow c\bar{c}[n]\{\gamma, g\}$  are related by crossing symmetry, as indicated below. As usual,  $d\sigma/dt$  is evaluated from the absolute square of the transition matrix element  $\mathcal{M}$  through multiplication with factors for flux, phase space, spin, and color, as

$$\frac{d\sigma}{dt} = \frac{1}{2s} \frac{1}{8\pi s} \frac{1}{4} \left(\frac{1}{8}\right)^i |\mathcal{M}|^2, \quad (4)$$

where  $i = 1, 2$  is the number of color-octet partons (gluons or ghosts) in the initial state.

The only non-vanishing ghost contributions read

$$\begin{aligned} |\mathcal{M}|^2(\gamma h \rightarrow c\bar{c}[{}^1S_0^{(8)}]h) &= \frac{24e^2 g_s^4 \langle \mathcal{O}[{}^1S_0^{(8)}] \rangle Q_c^2 su}{Mt(s+u)^2}, \\ |\mathcal{M}|^2(\gamma h \rightarrow c\bar{c}[{}^3P_0^{(8)}]h) &= \frac{32e^2 g_s^4 \langle \mathcal{O}[{}^3P_0^{(8)}] \rangle Q_c^2 su}{M^3 t(s+u)^4} [(2t+3u)^2 + 6s(2t+3u) + 9s^2], \\ |\mathcal{M}|^2(\gamma h \rightarrow c\bar{c}[{}^3P_1^{(8)}]h) &= \frac{32e^2 g_s^4 \langle \mathcal{O}[{}^3P_1^{(8)}] \rangle Q_c^2}{M^3 (s+u)^4} [u^2(t+u) - su^2 + s^2(t-u) + s^3], \\ |\mathcal{M}|^2(\gamma h \rightarrow c\bar{c}[{}^3P_2^{(8)}]h) &= \frac{32e^2 g_s^4 \langle \mathcal{O}[{}^3P_2^{(8)}] \rangle Q_c^2}{5M^3 t(s+u)^4} [3tu^2(t+u) + su(8t^2 + 21tu + 12u^2) \\ &\quad + 3s^2(t^2 + 7tu + 8u^2) + 3s^3(t+4u)], \\ |\mathcal{M}|^2(gh \rightarrow c\bar{c}[{}^1S_0^{(1)}]h) &= \frac{4g_s^2 \langle \mathcal{O}[{}^1S_0^{(1)}] \rangle}{3e^2 Q_c^2 \langle \mathcal{O}[{}^1S_0^{(8)}] \rangle} |\mathcal{M}|^2(\gamma h \rightarrow c\bar{c}[{}^1S_0^{(8)}]h), \\ |\mathcal{M}|^2(gh \rightarrow c\bar{c}[{}^3P_J^{(1)}]h) &= \frac{4g_s^2 \langle \mathcal{O}[{}^3P_J^{(1)}] \rangle}{3e^2 Q_c^2 \langle \mathcal{O}[{}^3P_J^{(8)}] \rangle} |\mathcal{M}|^2(\gamma h \rightarrow c\bar{c}[{}^3P_J^{(8)}]h), \\ |\mathcal{M}|^2(gh \rightarrow c\bar{c}[{}^1S_0^{(8)}]h) &= \frac{5g_s^2}{12e^2 Q_c^2} |\mathcal{M}|^2(\gamma h \rightarrow c\bar{c}[{}^1S_0^{(8)}]h), \end{aligned}$$

$$\begin{aligned}
|\mathcal{M}|^2(gh \rightarrow c\bar{c}[^3S_1^{(8)}]h) &= \frac{3g_s^6 \langle \mathcal{O}[^3S_1^{(8)}] \rangle}{4M^5 s t u (s+u)^2} [t u^2 (t+u)^2 (3t-u) + s u (-2t^4 + 2t^3 u \\
&\quad + 7t^2 u^2 + 4t u^3 + u^4) + s^2 (3t^4 + t^3 u - 4t^2 u^2 - 3t u^3 + u^4) \\
&\quad + s^3 (4t^3 + 7t^2 u - 2t u^2 - u^3) + s^4 (t^2 + 6t u - u^2)], \\
|\mathcal{M}|^2(gh \rightarrow c\bar{c}[^1P_1^{(8)}]h) &= \frac{g_s^2 \langle \mathcal{O}[^1P_1^{(8)}] \rangle}{e^2 Q_c^2 \langle \mathcal{O}[^1S_0^{(8)}] \rangle M^2} |\mathcal{M}|^2(\gamma h \rightarrow c\bar{c}[^1S_0^{(8)}]h), \\
|\mathcal{M}|^2(gh \rightarrow c\bar{c}[^3P_J^{(8)}]h) &= \frac{5g_s^2}{12e^2 Q_c^2} |\mathcal{M}|^2(\gamma h \rightarrow c\bar{c}[^3P_J^{(8)}]h), \tag{5}
\end{aligned}$$

where  $e = \sqrt{4\pi\alpha}$ , with  $\alpha$  being Sommerfeld's fine-structure constant, and  $g_s = \sqrt{4\pi\alpha_s}$  are the electromagnetic and strong gauge couplings,  $Q_c$  and  $m_c$  are the fractional electric charge and mass of the  $c$  quark, and  $M = 2m_c$ . By four-momentum conservation, we have  $s + t + u = M^2$ .

We now explain how the unpolarized and polarized results of Refs. [5, 6] may be recovered from the results of Ref. [2] in combination with Eqs. (4) and (5), considering  $\gamma g \rightarrow c\bar{c}[^1S_0^{(8)}]g$  as an example. The unpolarized and polarized results of Eqs. (A4) and (A5) in Ref. [6] are obtained from Eq. (4) by inserting

$$\begin{aligned}
|\mathcal{M}|_{\text{unpol}}^2(\gamma g \rightarrow c\bar{c}[^1S_0^{(8)}]g) &= \sum_{\xi_a, \xi_b = \pm 1} |\mathcal{M}|_{\xi_a, \xi_b}^2(\gamma g \rightarrow c\bar{c}[^1S_0^{(8)}]g) - |\mathcal{M}|^2(\gamma h \rightarrow c\bar{c}[^1S_0^{(8)}]h) \\
&\quad - |\mathcal{M}|^2(\gamma \bar{h} \rightarrow c\bar{c}[^1S_0^{(8)}]\bar{h}), \\
|\mathcal{M}|_{LL}^2(\gamma g \rightarrow c\bar{c}[^1S_0^{(8)}]g) &= \sum_{\xi_a, \xi_b = \pm 1} (-1)^{\xi_a \xi_b} |\mathcal{M}|_{\xi_a, \xi_b}^2(\gamma g \rightarrow c\bar{c}[^1S_0^{(8)}]g), \tag{6}
\end{aligned}$$

respectively, where  $|\mathcal{M}|_{\xi_a, \xi_b}^2(\gamma g \rightarrow c\bar{c}[^1S_0^{(8)}]g)$  may be gleaned from Eq. (A5) of Ref. [2] and  $|\mathcal{M}|^2(\gamma h \rightarrow c\bar{c}[^1S_0^{(8)}]h) = |\mathcal{M}|^2(\gamma \bar{h} \rightarrow c\bar{c}[^1S_0^{(8)}]\bar{h})$  is given in Eq. (5) above. As mentioned above, all ingredients entering  $|\mathcal{M}|_{LL}^2(\gamma g \rightarrow c\bar{c}[^1S_0^{(8)}]g)$  are contained in Ref. [2]. By crossing symmetry, we have

$$\begin{aligned}
|\mathcal{M}|^2(h\gamma \rightarrow c\bar{c}[^1S_0^{(8)}]h) &= |\mathcal{M}|^2(\gamma h \rightarrow c\bar{c}[^1S_0^{(8)}]h) \Big|_{t \leftrightarrow u}, \\
|\mathcal{M}|^2(h\bar{h} \rightarrow c\bar{c}[^1S_0^{(8)}]\gamma) &= |\mathcal{M}|^2(\gamma h \rightarrow c\bar{c}[^1S_0^{(8)}]h) \Big|_{s \leftrightarrow t}. \tag{7}
\end{aligned}$$

Similar relationships hold for the other partonic subprocesses involving two external gluons considered in Ref. [2].

In conclusion, we complemented the partonic cross sections for the inclusive production of prompt  $J/\psi$  mesons in polarized hadron-hadron, photon-hadron, and photon-photon collisions listed in the Appendix of Ref. [2] by providing the ghost contributions, which cancel in the cross section differences entering  $\mathcal{A}_{LL}$ , but contribute to absolute cross sections, including the unpolarized ones.

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